Math 126A Name Key

Exam 1

Spring 2015 Show All Work

1) Simplify each expression and eliminate any negative exponents. Assume that all letters denote positive numbers.

a) $\frac{24x^2y^{-2}}{6x^{-4}y^3} = \frac{4x^6}{y^5}$

b) $(27x^{-6}y^3)^{2/3} = 27^{2/3} \cdot (x^{-6})^{2/3} (y^3)^{2/3}$

$= 3^2 \cdot x^{-4} \cdot y^2$

$= \frac{9y^2}{x^4}$

2) Simplify the expression.

a) $\sqrt[3]{8x^4} = \sqrt[3]{2^3 \cdot x^3 \cdot x} = 2x \cdot \sqrt[3]{x}$

b) $\sqrt{75} - \sqrt{48} = \sqrt{25 \cdot 3} - \sqrt{16 \cdot 3}$

$= 5\sqrt{3} - 4\sqrt{3}$

$= \sqrt{3}$

3) Multiply and simplify $(x - 2)(x^2 + x + 3)$

$x(x^2+x+3) - 2(x^2+x+3)$

$= x^3 + x^2 + 3x - 2x^2 - 2x - 6$

$= x^3 + x^2 - 2x^2 + 3x - 2x - 6$

$= x^3 - x^2 + x - 6$

4) Factor the expressions completely

a) $2x^2 - 5x - 7 =$

$(2x-7)(x+1)$

b) $9x^2 - 4 =$

$(3x-2)(3x+2)$

c) $x^3 - 8 =$ $(x-2)(x^2+2x+4)$

d) $2x^{3/2} + 3x^{-1/2} =$ $x^{-1/2}(2x^2+3)$

5) Perform the indicated operation and simplify

a) $\frac{x^2-x-6}{x^2+3x} \div \frac{x-3}{x+3} = \frac{(x-3)(x+2)}{(x+3)x} \cdot \frac{x+3}{x-3} = \frac{x+2}{x}$

invert and multiply

(Perform the indicated operation and simplify)

b) $\frac{1}{x^2+6x+8} - \frac{3}{x+4}$

$= \frac{1}{(x+2)(x+4)} - \frac{3}{x+4} = \frac{1}{(x+2)(x+4)} - \frac{3}{x+4}\cdot\left(\frac{x+2}{x+2}\right)$

$= \frac{1}{(x+2)(x+4)} - \frac{3(x+2)}{(x+2)(x+4)} = \frac{1-3x-6}{(x+2)(x+4)} = \frac{-3x-5}{(x+2)(x+4)}$

6) Rationalize the denominator of $\frac{3}{4+\sqrt{2}} = \frac{3}{4+\sqrt{2}}\cdot\frac{4-\sqrt{2}}{4-\sqrt{2}} = \frac{3(4-\sqrt{2})}{16-(\sqrt{2})^2}$

$= \frac{3(4-\sqrt{2})}{16-2} = \frac{3(4-\sqrt{2})}{14}$

7) Phyllis invested 15,000, a portion earning a simple interest rate of 4.5% per year and the rest earning a simple interest rate of 5% per year. After 1 year the total interest earned on these investments was $723. How much money did she invest at each rate?

x = amount at 4.5%

y = amount at 5%

$x+y = 15{,}000 \rightarrow y = 15{,}000 - x$

$.045x + .05y = 723$

$.045x + .05(15000 - x) = 723$

$.045x + 750 - .05x = 723$

$-.005x = -27$

$x = 5400$

$y = 9600$

Phyllis invested $5400 at 4.5% and $9600 at 5%

Final answer

8) Solve the following equations

a) $\frac{2x-1}{x+3} = \frac{4}{5}$

$5(2x-1) = 4(x+3)$

$10x - 5 = 4x + 12$

$6x = 17$

$x = \frac{17}{6}$

b) $\frac{4}{x-1} + \frac{2}{x+1} = \frac{26}{x^2-1}$ $\quad$ Lcd $= (x-1)(x+1) = x^2-1$

$(x+1)(x-1)\left[\frac{4}{x-1} + \frac{2}{x+1}\right] = (x+1)(x-1) \cdot \frac{26}{x^2-1}$

$4(x+1) + 2(x-1) = 26$

$4x + 4 + 2x - 2 = 26$

$6x + 2 = 26$

$6x = 24$

$x = 4$

9) Find all solutions of the equation.

a) $x^2 - 5x - 6 = 0$

$(x-6)(x+1) = 0$

$x - 6 = 0$ or $x + 1 = 0$

$\boxed{x = 6 \text{ or } x = -1}$

b) $x^2 - 6x + 3 = 0$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-(-6) \pm \sqrt{36 - 4 \cdot 1 \cdot 3}}{2 \cdot 1} = \frac{6 \pm \sqrt{36-12}}{2}$

$= \frac{6 \pm \sqrt{24}}{2} = \frac{6 \pm \sqrt{4 \cdot 6}}{2} = \frac{6 \pm 2\sqrt{6}}{2} = \frac{2(3 \pm \sqrt{6})}{2}$

$= 3 \pm \sqrt{6}$